# Interactions between abstract actions and apparent distance

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**Abstract**

Perceptions influence the way we act in our environment based upon judgments assessing required efforts to perform an action and the availability and demand for immediate action on an object (Proffitt, 2006B). Social and physical anxiety has been shown to distort perceptions of depth and perceptions of object size (Stefanucci et al., 2008; Cañal-Bruland et al., 2010). Relatively little work, however, has explored the potential role of depth perception in abstract reasoning tasks (Landy & Linkenauger, 2010). In Experiment 1, the relationship between depth perception and the order of actions taken to simplify arithmetic expressions was investigated by manipulating apparent distances of arithmetic operations of high and low syntactic precedence. When the high precedence operations appeared to be closer to the participant, expressions were solved more quickly than when low precedence expressions appeared to be closer. Experiment 2 explored the whether the affordance of abstract actions conversely impacted perceived distance by asking participants to make distance judgments to multiplication and addition operations. Experiment 2 found no impact of anxiety about mathematics on perceived distance. However, effects resulting from condition assignment were found to influence perceived distances, as well as solving strategy. We interpret results in terms of attention, which we speculate plays a key role moderating both ordering behavior and perceived distance.

**Keywords:** Perception; Anxiety; Attention

## Introduction

The basic purpose of perception is to guide sensible action. Specifically, perception functions as a tool that informs and guides actions (Proffitt, 2006A). Perception involves consideration of the necessary efforts required to perform an action, which in turn may bias judgments of object size and proximity. For instance, perceived distance of an object is more than simple metric distance. Participants carrying a heavy backpack estimate the slant of hills to be steeper than those not wearing backpacks (Proffitt, 2006B). Likewise, simply intending to act on an object could make it seem closer Witt & Proffitt (2005). Witt & Proffitt (2005) found that athletes playing well reported a softball ball as being bigger than its actual size and therefore closer. Their ability to hit the ball was correlated with their batting average, demonstrating the relationship between perceptions of object size and performance when acting on that object. Perceptual phenomena like these indicate that perception relates the body and goals to the opportunities and costs of acting in the physical environment.

Although the connection between concrete physical action capabilities and depth perception is well documented, much less work has explored the relationship between perceived depth and non-concrete behaviors. There is evidence that anxiety also influences perception (Proffitt, 2006A). For instance, participants with a fear of heights judge hills to be steeper than do those who are less afraid (Stefanucci et al, 2008). According to Teachman et al (2008), fear of heights is associated with perceptual biases in judging heights, implying that an individual's emotional state influences what is seen, perhaps because it affects perceived costs, such as the cost of falling down a steep hill.

Anxiety not only distorts perception in the sense that distances appear farther and angles steeper, but also by altering the appearance of an object's size. Cañal-Bruland, Pijpers, & Oudejans (2010) studied the relationship between anxiety and depth perception, while also taking into consideration perceptions of object size. Participants were asked to throw darts at a target from a position on a rock wall. Cañal-Bruland, et al. found that the low anxiety group performed better and saw the target as bigger. However, these findings are limited to perceptions related to actions in the physical world.

Even less is known about cases in which the action itself is abstract or non-physical. In the case of abstract calculation, since all actions are in principle equally easy to perform, the logic of perceived energetic cost per se does not predict any relationship between depth and intentions to act. However, mathematical rules specify that certain operations are to be performed before other others. Particularly relevant for the studies reported here are the order of precedence rules, which require that multiplications be executed before additions., As a result, some operations demand action before others, making, it possible to vary the relative availability of actions. Furthermore, since the actions do not depend on reach distance, any action lies within the action boundary (Fajen, 2005). Landy & Linkenauger (2010) found a relationship between the availability of computational actions in compound arithmetic expressions and judgments of depth. The study explored judged depth of terms, and indeed found that participants in a forced-choice task preferred to align depth and precedence; furthermore, in a face-vase illusion in which sub-expressions of mathematical equations were superimposed onto a face-vase illusion, when the times sign was over the vases, participants reported seeing the faces less often than when the times sign was over the faces. These perceptual effects can be explained by the affordances of immediate action associated with the times sign as a result of taking precedence when using order of operations in solving arithmetic expressions. The current work builds on Landy & Linkenauger (2010) by considering whether perceived (rather than judged) distance interacts with the availability of concrete action

Visual attention also plays a significant influence on action-specific perception and performance. In their study, Cañal-Bruland, Zhu, van der Kamp, & Masters (2011) explored this relationship through a golf-putting task that manipulated the target-directed visual path. They found that participants receiving full visual access to the target and who putted more successfully estimated the target circle to be bigger than their less successful counterparts. Thus, the results of this study have shown that visual attention influences perceptions object size, but only for objects of an intended action. The relationship between anxiety and depth perception is abstract when attentional influence is taken into consideration. The current study aims to further explore attention as a moderating variable in the relationship between math anxiety and depth perception.

Math anxiety is defined by a strong tendency to avoid math, which leads to lower competency levels in math compared to those without math anxiety (Ashcraft, 2002). Hoffman (2010) posits that anxiety is a common impediment to learning in college students. Anxiety has shown to impede working memory processes involved with problem-solving efficiency, especially for women. Implications from prior research suggest that more needs to be done in order to understand the perceptual difficulties associated with math anxiety, especially if research can identify attention as a target for future interventions.

The purposes of the current studies were as follows: first, to establish whether the effect on judgments of apparent distance in pictures reported by Landy & Linkenauger (2010) generalized to perception in physical situations and second, to evaluate the influence of anxiety in distorting perceptions of depth in abstract situations.

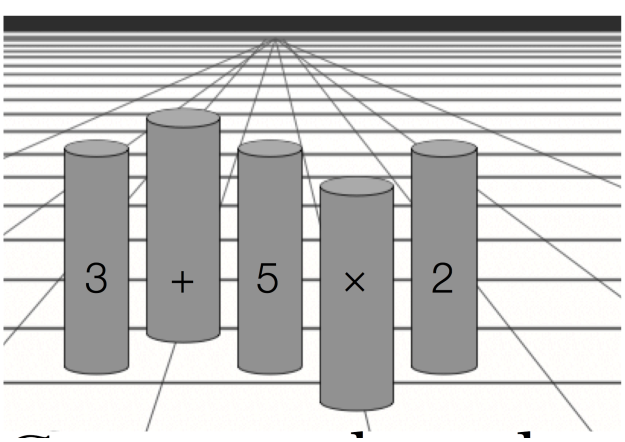


Figure 1: The stimuli used in Experiment 1. In this congruent problem, the times sign (which is high precedence) appears to be closer to the subject.

# Experiment 1

## Participants & Procedure

Thirty members of the University of Richmond community were given partial course credit in exchange for participation.

Participants sat in front of a computer. Participants were shown 128 simple arithmetic problems, and were instructed to state the solution to the problems out loud. Before beginning, participants were reminded of the order of operations through direct instruction and an example, and were explicitly instructed to ignore any irrelevant images or visual structure. Responses were recorded by microphone and analyzed using CheckVocal (Protopapas, 2007).

All trials involved single digit operands, and were of one of two forms: $a+b\times c$ or $a\times b+c$. Correct solution values ranged from 13 to 76.

The first 8 trials were practice trials. In these trials, problems were presented against a white background. In the remaining 120 trials, problems were presented against a background image intended to affect the apparent depth of the operators and operands. In this image (see Figure 1), stimulus problems appeared to be placed on a set of pillars, which varied in whether the left operator appeared closer (*left-closer* condition), or the right (the *right-closer* condition).

We hypothesized that problems would be solved more easily when the high-precedence operation appeared to be closer to the participant (called *congruent* problems). Following comparable results in the manipulation of spacing cues (Landy & Goldstone, 2010) we expected the alignment of precedence and distance cues to selectively influence order of operation reversals and operation errors (e.g., performing a multiplication instead of an addition), and to affect correct-trial response time.

## Results

Because trial RTs were substantially non-normal the median response time for each condition was calculated for each subject, and subjected to a standard ANOVA. Pillar and operation structure functioned as independent variables (see Table 1). Neither main effect was significant (pillars: $F(1,28)=0.04$, $p\sim0.85$, operations: $F(1, 28)=1.2$, $p\sim0.3$); the interaction was significant and in the predicted direction ($F(1,28)=10.4$, $p<0.01$).

Mean accuracy was 0.90 (Min=0.79, max=1.0). Participants made very few order of operations errors and operation errors: together, these made up just 2.9% of responses. There was no difference between problems that aligned pillars and operation structure and problems that misaligned them. The error rate was 0.1 (SE=0.01) on incongruent problems, and 0.06 on congruent problems (SE=0.01) on congruent problems. A logistic regression over operation order and pillar structure revealed neither a main effect (operations: z=0.4, pillars: z=0.16, all p's > 0.5), nor an interaction between the two (z=0.5, p~0.6). Results

Table 2: Mean response time for accurate trials in Experiment 1 (Errors are standard errors).

| Leftmost operation | Closer Pillar | |
|---|---|---|
| | Left | Right |
| Plus | 2619 | 2516 |
| | (127) | (111) |
| Times | 2465 | 2554 |
| | (129) | (131) |

were nearly identical when just order and operation errors were considered: The mean rate of operation errors and order errors was 0.03 (SE=0.007) and 0.028 (SE=0.007) for congruent and incongruent problems, respectively; there was no difference between these by a logistic regression over pillar structure and operation (pillar z = -1.1, operation order = -0.1, interaction z = -0.5, all *p*'s > 0.61).

## Discussion

Apparent distance did, as predicted, influence the execution of basic arithmetic problems. Despite the equal availability and readability of all terms in these problems (i.e., all problems were presented on identical local backgrounds, at identical sizes), problems in which high precedence problems were apparently nearer to the participant were solved more quickly than those in which the reverse was true. This suggests that reasoners use cues about physical structure when making abstract decisions such as operation ordering. Experiment 1 indicates a bidirectional relationship between apparent distance and arithmetic ordering.

In other similar work (e.g., Landy & Goldstone, 2010), we have typically observed both accuracy and response time effects. One difference between those and these was that the rate of errors in this study was very low overall (just 10%), and the rate of errors expected to be related to operation order was even lower (just 2.9% overall). The population of students in the current study was generally efficient at basic arithmetic. This fact, together with the direct reminder to follow the order of operations, may have shielded participants from making many direct strategic errors when solving the expressions.

# Experiment 2

Experiment 2 explored the relationship between perceived distance, action affordance, and experienced anxiety in an abstract domain--arithmetic. Participants judged the distance to specific symbols inside an arithmetic expression, which either did or did not afford immediate action. We hypothesized, first, that participants who knew the order of operations would judge times signs as closer than plus signs, since in unparenthesized expressions with multiple operations multiplications afford more immediate action than additions do. We explored whether demonstrations that anxiety distorts perceptions of action affordances would generalize to abstract actions and situations.

## Participants

The 32 participants of this study were from the Richmond community between the ages of 18 and 30. They were recruited through flyers posted around the University of Richmond campus, as well as, through weekly online campus announcements.

## Apparatus and Materials

**Experimental Room**. The room was set-up with two identical tables (137 cm x 76 cm) joined at a corner, forming an L-shape. The participant's chair was positioned at the corner of these tables facing the wall. Both tables were positioned approximately 23 cm from the wall. The front legs of the chair were positioned 12 inches from the front outer-legs of the table.

**Boards and Measuring Key**. Expressions were displayed on boards made of a white foam poster board, cut into rectangles (45.7 cm x 30.5 cm) and supported by 30.5 cm easel backs. The boards angled slightly, so that the top was slightly further from the participant than the bottom.

A total of eight boards were made, one for each arithmetic expression. The boards were placed at distances both within and out of reach to the participant. Measured from the back of the chair, expressions were placed at four distances: 80cm, 40cm, 110cm, and 50cm.

**Questionnaires**. Participants completed the Abbreviated Math Anxiety Scale (Hopko et al., 2003) and WRAT-3 assessment (Roberston, 2010), as well as a questionnaire assessing each participant's particular math background and interests.

## Procedure

Participants sat in a chair facing the corner of the two tables. To familiarize the participant with the procedure, a warm up trial consisting single row of five dots was presented on the right-hand table at a distance of 80cm. Participants were informed that following the warm-up, the board would display arithmetic expressions instead of dots. Because we were interested in the potential long-term influence of operation ordering practice on distance perception, participants were not instructed in a particular solving method. In particular, participants were not reminded of the order of operations, but were simply asked to solve the problems as they usually would, and to do their best.

Participants were instructed to estimate the distance from their sternum to the center dot. Participants indicated their estimate by extending the tape measure along the left-hand table, with the blank side of a tape measure up. This method allowed the participant to have full control over the estimation process, while blinding them to the actual measurement. When providing their estimate, participants were not permitted to use the tape measure on the table displaying the board, nor were they allowed to reach out and touch the board.

During actual experimentation, the procedure followed in the same manner as the warm-up, substituting arithmetic expressions for the row of dots. Participants viewed four expression pairs displayed on boards set at one of four distances (80cm, 40cm, 110cm, 50cm) from the edge of the table closest to the participant. We repeated the procedure for each participant and with variance in the complexity of the math problems: four multiplication and four addition, with two easy and two complicated in each. Problems were paired, so that for each problem a participant solved while focusing on a multiplication sign, they later solved a problem identical except for the substitution of additions for multiplications. For example, if given the problem 5+4x3+7, we asked them to focus on the multiplication sign when giving their estimate. Immediately following their estimate, they proceeded to solve the expression aloud, step-by-step.

The order of distances was fixed, but the problem presentations were counterbalanced in a Latin Square design, leading to a total of eight conditions (see *Table 1*).

Table 2: Stimuli for Experiment 2 (Condition 1)

| Expression | Center Operation | Distance (cm) | Difficulty |
|---|---|---|---|
| 3 x 5+ 2 x 7 | Add | 80 | Easy |
| 1/3 x 4 + 2/3 x 1/2 | Add | 40 | Difficult |
| 5 x 8 + 2 x 3 | Add | 110 | Easy |
| 1/4 x 1/6 + 5 x 2/3 | Add | 50 | Difficult |
| 3 + 5 x 2 + 7 | Mult | 80 | Easy |
| 1/3 + 4 x 2/3 + 1/2 | Mult | 40 | Difficult |
| 5 + 8 x 2 + 3 | Mult | 110 | Easy |
| 1/4 + 1/6 x 5 + 2/3 | Mult | 50 | Difficult |

Each expression pair was set at the same distance. Thus, if 5+4x3+7 was set at 50 cm, then 5x4+3x7 was also set at 50 cm. Between trials the participant was given a packet of mazes and instructed to complete as much of it as they could while the experimenter set up the next trial.

Following the distance perception session, the math anxiety level and abilities of each participant was assessed with a basic mathematics proficiency test, WRAT-3. Each participant was given 10 minutes to complete as much of the packet at they could, without returning to the problems they skipped. After the WRAT-3, participants completed the Abbreviated Math Anxiety scale.

## Results

Distance estimates (see Table 3) were subjected to an ANOVA using operation structure and problem difficulty, within-participants factors, and math anxiety and precedence behavior as between-participant factors. Participant precedence behavior was coded by the experimenter as either the correct use of order of operations or the incorrect use. Participants who did not use the order of operations correctly tended to use other strategies, such as adding from left to right, or computing sums before products. It was typically difficult to discern what strategy a participant had used, and some participants tended to shift strategies over the course of a trial. To be clear: differences in operations based on precedence strategy are expected only among subjects who apply correct order of operations. There was no significant main effect of math anxiety on distance estimates ($F(1,30)=.02$, $p\sim0.9$) or in the interaction between math anxiety and problem difficulty on distance estimation ($F(1, 30) = 1.10$, $p > .05$), There was, however, a significant interaction between the operation in focus and precedence behavior ($F(1, 30)=5.0$, $p<0.05$), such that operations which were treated as high precedence yielded lower distance estimates. A follow-up analysis considering just participants who correctly used order of operations revealed a significant main effect of focal operation ($F(1, 17) = 4.87$, $p < .05$). There was no significant interaction for who did not obey standard precedence rules; numerically, this group tended to estimate the plus signs as closer than the times signs ($F(1, 12)=3.1$, $p\sim0.11$).

Post-hoc analyses revealed a possible confound of the counterbalance condition, in that the condition in which the participant was randomly assigned seemed to influence whether or not they estimated the plus or times signs to be closer and the strategy used to solve the expressions. Specifically, those who received the conditions presenting the times sign as the operation in focus first tended to use order of operations when solving the expressions (87.5% correct precedence behavior), whereas those receiving the plus sign as the operation in focus tended not to (31% correct precedence). The effect of first operation on precedence behavior was significant by Fisher's exact test ($p<0.01$); however, since this effect was not predicted ahead of time, it should be interpreted cautiously. To ensure that the main analyses were not affected by an overall linear trend in distance judgments across trials, the data were reanalyzed using problem order as a covariate; results were in all ways similar to those reported above.

Table 3: Mean (standard error) distance underestimated distance to central operations in Experiment 2.

| Central operation | Precedence Behavior: Correct | Precedence Behavior: Reversed or Left/Right |
|---|---|---|
| Plus | 14.2 (1.7) | 17.2 (1.4) |
| Times | 16.8 (1.9) | 15.6 (1.4) |

### Discussion

Although our hypothesis that math anxiety would lead to overestimation of distances was not supported, our findings support the prediction that those who knew the order of operations would perceive the times sign to be closer. However, it is quite possible that null findings could be a result of such a small sample size. Taken together with the results from Experiment 1, high precedence operations afford immediate action and therefore, may appear to be closer. The effect of the experimental conditions suggest that attention may serve as a moderating variable between depth perception, anxiety, and the perceived efforts to enact an action.

## General Discussion

Two experiments verified and extended the basic findings of Landy & Linkenauger (2009), demonstrating a bidirectional relationship between perceived distance and effective arithmetic syntax. While prior results indicated a metaphorical relationship between distance and precedence, Experiment 2 here demonstrated that participants who correctly apply the order of operations also estimate the actual physical distance to high precedence sign as smaller than that to a low precedence sign. Furthermore, these are the first results to demonstrate that (simulated) physical distance affects the application of abstract formal operations.

Both of these phenomena are familiar in interactions with physical objects. Intentions and object affordances both impact perceived distance (Proffitt, 2006B). We know of no research directly exploring the impact of apparent distance on action selection, but it seems quite likely that actions with physical objects are selected in part on the basis of perceived proximity. In the current experiments, however, the relevant action is itself abstract, and the ease of adding and multiplying does not depend in any obvious way on physical proximity. The interpretation of interactions between apparent distance and abstract actions is thus less clear than with concrete objects. A dramatic reading might hold that explicit distance perception tracks the perceived difficulty of engaging in behavior—that is, that explicit perceptions of distance are fundamentally less concrete and more abstract than has previously been supposed. The fact that we did not find any sign of a relationship between math anxiety, problem difficulty, and perceived distance speaks against such a dramatic conclusion.

A more plausible interpretation is that abstract procedures, such as calculation are executed via systems normally devoted to perception and action (Landy & Goldstone, 2007, 2009; Goldstone, Landy, and Son, 2010; Landy, Allen, and Anderson, 2011). On this "Rigging Up Perceptual Systems" account, learning to engage in formal operations, such as operation ordering often involves adapting a pre-existing perceptual-motor system that already performs computations roughly appropriate to the to-be-learned content. Previously identified systems include the use of perceptual grouping and attention to perform operation ordering (Landy & Goldstone, 2007, 2010; Goldstone et al 2010); it may be that some individuals implement operation ordering via distance perception mechanisms, learning to treat sub-expressions which should be ignored as farther away, and so leveraging powerful machinery that produces distance judgments to automatize routine computation.

It also seems possible that differences in perceived distance may simply be a result of focused attention: that is, it is possible that simply attending to an object increases its apparent proximity. Though we know of no direct demonstration of this possibility, attention is thought to affect other aspects of perception, such as contrast (Treue, 2004), apparent speed of motion (Turatto et al, 2007), and apparent size (Anton-erxleben & Treue, 2007; but see Schneider, 2008). Attentional focus may also influence perceived distance. This explanation provides a natural account for the influence of precedence judged distance and figure and ground perception in the face-vase illusion previously demonstrated by Landy & Linkenauger (2009). Furthermore, attention is thought to be necessary for action-specific effects on perceived distance (Cañal-Bruland et al., 2011) Finally, though it should be interpreted with caution, the unpredicted relationship between initial focal operation and both distance and precedence behavior is also compatible with an attention-based interpretation. It may be that asking people to attend to a particular sign influenced both ordering behavior and perceived depth.

### Conclusions

In our experiments, we did not find any effect of perceived difficulty on estimated distance, either when problem difficulty varied, nor based on personal skill or arithmetic self-efficacy. Of course, null results such as these must be interpreted with caution; nevertheless, there is little indication here of a very tight analogy between estimations of abstract difficulty and perceptions of physical distance.

On the other hand, we found substantial bidirectional influences between order of operations and perceived depth, suggesting that the relationship between action ordering and depth is not restricted to concrete behaviors, but is also involved in abstract actions as well.

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