Why Spatial-Numeric Associations Aren't Evidence for a Mental Number Line

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Abstract

In the well-known SNARC effect, people are shown to be faster at responding to relatively large numbers with their right hands, and to relatively small numbers with their left hands, on magnitude-irrelevant tasks. It is typically assumed that both the SNARC effect and the Simon effect—in which right-hand responses are facilitated when stimuli appear on the right-hand side—are explained by spatial mappings of stimulus to response; where the former is the result of representing numbers spatially (i.e. along a mental number line) and mapping right-hand responses to large numbers and left-hand responses to small numbers, and the latter is the result of responding to stimuli on the right side of visual space with the right hand and vice-versa. However, the principle of polarity correspondence can also account for these findings; this theory asserts that in binary representations of dimensions, one pole is dominant and tasks are facilitated when dominant stimulus values are mapped to dominant response values. In previous studies, dominance and spatial mapping have been confounded. The current paper uses non-spatial responses (spoken "yes" and "no" responses) to consider whether polarity correspondence can account for the SNARC effect.

Keywords: Mathematical Cognition; SNARC effect; Stimulus-Response Compatibility; Simon Effect.

Introduction

The number keys on a typical American or European keyboard are laid out with small numbers on the left, and larger numbers on the right. On a telephone keypad, the numbers are arranged with small on the left, and large on the right. In school students are taught to reason using a "number line", which arrays the numbers in linear order, with (guess what?) small numbers on the left, and large numbers on the right.

Dehaene, Bossini, and Giraux (1993) demonstrated that people respond to small numbers in a spatially asymmetric way. Even when the task requires minimal processing of numerical magnitude (as in Dehaene et al. (1993) and in the current paper, where the task was to determine whether the number was even or odd), people respond more rapidly to large numbers with their right hand, and more rapidly to small numbers with their left hand. The direction of this effect (termed the Spatial-Numeric Association of Response Codes, or SNARC effect) seems to correspond to typical writing direction (Zebian, 2005), suggesting that the association is acquired (but see Hubbard et al., 2004).

The usual explanation of the SNARC effect takes the system of linearizing numbers utilized in keyboards and in number lines and cognitively reifies it, by assuming that numbers are represented spatially, i.e., along a mental number line. Thus, the mental representation of '9' includes the property "right of 2". Note that the posited code is relative—9 is represented as being more right than, say, 2, but leftward of 20.

The SNARC effect, then, results from the fact that the representation of a large number overlaps more with the representation of a right-hand response than it does with that of a left-hand response. This account—which we will call a 'representational overlap account—has been used to explain a wide variety of effects relating to sides of space and numerical magnitude (e.g., Fischer, 2001; Casarotti et al., 2007). A very similar representational overlap account has been used to explain the well-known Simon effect (Simon, 1990). In a typical experiment illustrating the Simon effect, participants respond to non-spatial properties of simple stimuli, such as color, using a right-hand and a left-hand response. Responses are facilitated when the side of correct response corresponds to the side of presentation, even though side of presentation is formally irrelevant to the task. As in the mental number-line account of the SNARC effect, the stimulus-response compatibility arises from the overlapping spatiality in representation of stimulus and response.

Recently, a different kind of account has been proposed that explains the SNARC effects along with the Simon

effect under a framework that does not involve representational spatial overlap. With this account, called Polarity Correspondence (Cho and Proctor, 2006; see also Chase and Clark, 1971; Seymour 1973), both the Simon effect and the SNARC effect result from a fundamental principle of stimulus-response mapping. This principle posits that in binary representations of dimensions, across both stimuli and response properties, one value of the dimension (here called the *dominant value*), is generally more available than the other. Operationally, dominant values can be detected because people are generally faster to evaluate dominant values than non-dominant values. Linguistically, furthermore, non-dominant values are often marked, and come second in comparatives. It is typical to use values from the dominant end of a dimension as examples of the dimension. In the introduction of this paper, for instance we notice that we followed this typical practice by principally using the values "right", "large", and '9' as default examples of the spatial and numerical dimensions, non-dominant terms appeared only immediately after a dominant term**.** The principle of polarity correspondence, then, is this:

Polarity Correspondence Principle: Tasks will be facilitated whenever dominant values of varying stimulus dimensions are mapped onto dominant response values.

The amount of facilitation should correspond to the degree to which the dimension in question is encoded in the process of accomplishing the task.

This principle explains the SNARC effects as a special case: because large is positive with respect to magnitude and right-hand responses dominate left-hand responses, responding should be facilitated when large numbers align with right-hand responses. The principle also explains a wide swath of related literature such as the vertical SNARC (Ito and Hatta, 2004) and Simon and vertical Simon (e.g., Lippa and Adam, 2001) effects, the MARC effect (Willmes & Iversen, 1995), the spatial-musical association of response codes (Lidji et al, 2007), and many others (e.g., Fisher, 2001). The principle also explains the role of pole alignments in children's construction of opposites (Sheya, 2003).

The Polarity Correspondence Principle is not without its weaknesses, however. Unlike the mental number line account of SNARC, it gives no obvious causal explanation of why compatible value dominance should yield non-additive facilitation. Why should mapping dominant stimulus properties onto non-dominant response values yield interference? The polarity correspondence account is silent, taking such interference as an assumption. Moreover, it provides no general account of why binary dimensions should have a dominant value at all (though see Cho and Proctor, 2007, for one factor). What properties of the dimension or of human experience should make one value more available than another?

A second problem for polarity correspondence is that while it does predict many previously reported stimulus-response compatibility effects, it also predicts a host of novel effects, some of which may seem otherwise to be quite unmotivated. For any pair of salient stimulus and response dimensions, there should be a potential mapping interference.

If the polarity correspondence account is correct, then responses to number should benefit from mappings onto dominant responses, regardless of whether those responses are spatial. On the other hand, if the SNARC effect is really the result of a mental number line, then there is no good reason to expect a comparable effect along non-spatial dimensions. The goal of this paper is to un-confound the role of spacing and polarity dominance in the SNARC effect by replicating one of Dehaene et al.'s original experiments demonstrating the existence of the SNARC effect, but replacing left- and right-hand responses with a responses along a non-spatial but asymmetric response dimension (spoken 'yes' and 'no' responses). The polarity correspondence principle predicts that the results will be analogous to the SNARC result: participants should be faster to respond when responding 'yes' to larger numbers, and 'no' to smaller numbers. While the mental number line theory makes no prediction about this case, to the extent that the correspondence principle is supported, the mental number line is not useful in explaining the original SNARC effect.

In this experiment, there are actually (at least) two relevant stimulus dimensions: magnitude and parity. We will explore the data then, by looking for both a spatial-response alignment and a parity-response alignment.

Experiment Methodology

The experiment consisted of three sectioned tasks, with the same materials and methodology. Only the instructions differed between sections. In each task, the participant looked at a number, made a simple decision about that number, and then made a response. In the first two tasks, the decision was about the parity of the number, and the response was either spatial (left-right button-press) or valued ('yes' versus 'no', spoken aloud). In the third task, the verbal 'yes'-'no' response was paired with an explicit magnitude decision.

Method

Participants 33 undergraduates at the University of Illinois received partial course credit in exchange for participation in the experiment.

Materials In each section of the experiment, participants viewed a new instruction set, and then saw and responded to 160 stimuli. Each stimulus consisted of a number in the range 0-9, displayed centrally as an Arabic numeral on a computer monitor for 750ms. After this time, the stimulus was removed and replaced by a blank screen for 1750ms,
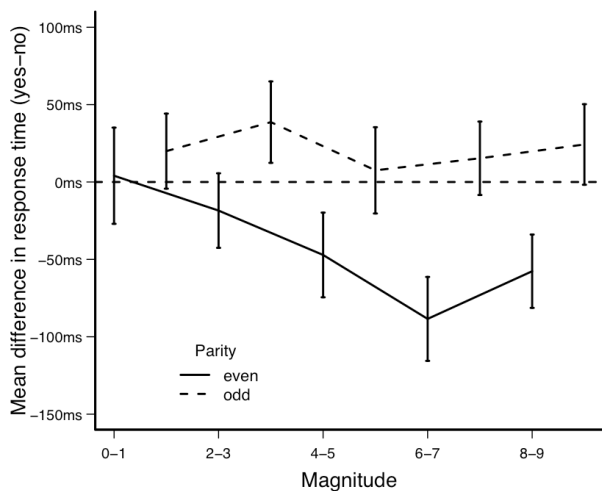
Figure 1: Mean difference between per-participant median response times for "yes" responses and "no" responses, collapsed across digit pairs, on the spoken parity task (all errors are standard errors).

and then a new stimulus was presented. In all three sections, instructions emphasized both speed and accuracy.

Each section was divided into four blocks of 40 stimuli: each block of 40 contained 4 instances of each of the digits between 0 and 9, presented in a random order. These experiments were programmed in DMDX (Forster and Forster, 2003), and analyzed using CheckVocal (Protopas, 2007).

Procedure

In the first section, participants were asked to respond to the parity of the presented digit. Responses were the words "yes" and "no" spoken aloud. Instructions alternated by block between asking participants to respond "yes" to even digits and "no" to odd digits, and asking participants to respond "no" to even digits and "yes" to odd digits. In all three sections, the initial mapping was counterbalanced across subjects.

In the second section, participants were asked to respond to parity again. However, unlike section one, participants in this section responded spatially. Now instructions alternated by block between asking participants to respond by pressing the right shift key to even digits and the left shift key in response to odd digits, and asking participants to respond with the left shift key to even digits, and the right shift key to odd digits. Participants were instructed to keep their left index finger on the left shift key, and their right index finger on the right shift key at all times.

The order of the first two sections was counterbalanced; since the intention of the task is that magnitude be implicit, the explicit magnitude task was always presented last.

In the final section, participants were asked to respond to numerical magnitude. Instructions alternated between asking participants to respond "yes" to numbers 5 and larger and "no" to numbers 4 and smaller, and asking participants

to respond "no" to numbers 5 and larger, and "yes" to numbers 4 and smaller.

Results

Accuracy was much lower for the number zero (77%) than for all other items (95.4%-97.5%), a significant difference ($F(9, 288)=3.26$, $p<0.001$). Several participants asked the experimenter whether to treat 0 as even. For this reason, responses to zero stimuli were removed from response time analyses in sections 1 and 2. Including these items did not affect the character of the results.

Section 1: Spoken Parity

Following the analysis of Dehaene et al. (1993), median RTs for correct responses were computed for each target number, response code, and subject. These data were analyzed in a 2 (parity) × 2 (expected response: 'yes' or 'no') ANOVA using magnitude as a linear variable. Following, Dehaene, 1 magnitude was collapsed across digit pairs: 0-1, 2-3, 4-5, 6-7, 8-9 (All results were also run without collapsing across digit pairs. Both the qualitative patterns and the statistical significance were unaffected; recall that zero items were not included).

Figure 1 displays the mean benefit for a 'yes' response in accurate response times as a function of number magnitude. Since the vertical axis is a difference, negative values indicate that participants responded more quickly on trials for which 'yes' was the correct response than on 'no' trials (when they answered correctly). It can be seen that, particularly for even numbers, the effect of response type interacted with numerosity.

The full ANOVA revealed both a SNARC and a MARC effect: magnitude interacted with correct response ($F(1,32)=12.05$, $p<0.01$), as did parity ($F(1,32)=13.12m$, $p<0.001$). There was a tendency, apparent in Figure 1, for the effect to be stronger for even than for odd numbers, but this tendency was only marginally significant ($F(1,32)=3.4$, $p\sim0.075$).

The primary goal of this study was to establish the existence of a relationship between stimulus magnitude and verbal response valence. Since it was possible that spatial responding on the manual task might affect the verbal task, we also analyzed just the participants who performed the verbal task first. As in the full set, there was a significant interaction between magnitude and correct response: ($F(1,16)=12.9$, $p<0.01$), and also between parity and correct response: ($F(1,16)=22.7$, $p<0.001$).

Section 2: Manual Parity

Again, median RTs for correct responses were computed for each target number, response code, and subject. These data were analyzed in a parity × expected response (left or right) × magnitude ANOVA. Figure 2 displays the mean difference in accurate response times as a function of number magnitude. As in Section 1, there was a significant interaction between parity and response type
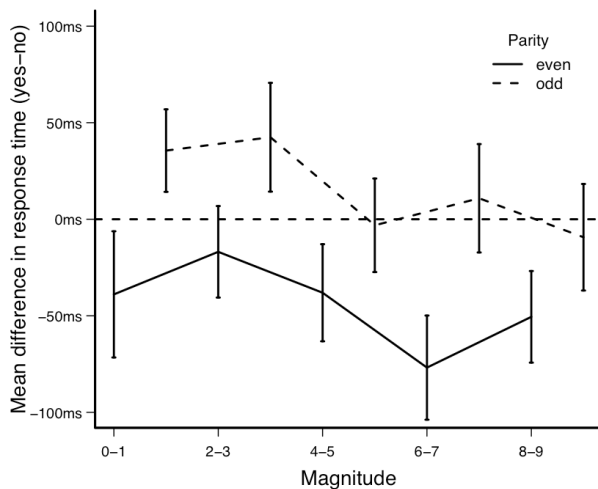
Figure 2: Mean difference between per-participant median response times for "yes" responses and "no" responses, collapsed across digit pairs, on the manual parity task.

(F(1,32)=4.7, p<0.05), as well as an interaction between magnitude and response (F(8.07), p<0.01).

An ANOVA comparing the magnitude-response and parity-response interactions across section revealed that there were significant 3-way effects of order: both the magnitude-response interaction and the parity-response were larger in the modality tested first (magnitude-response: F(1,30)=9.96, p<0.01, parity-response: F(1,30)=11.13, p<0.01). neither effect differed between section (magnitude-response: F(1,30)=0.42, p~0.5, parity-response: F(1,30)=0.02, p~0.89).

Section 3: Spoken Magnitude

Finally, each subject completed an explicit magnitude task, in which 'yes' and 'no' responses mapped onto whether the number was less than or equal to 4, or greater than or equal to 5. Again, median RTs for correct responses were computed for each target number, response code, and subject. These data were analyzed in a parity × correctness × magnitude ANOVA. Figure 4 displays the mean difference in accurate response times as a function of number magnitude.

The full ANOVA revealed a significant interaction of correct response and magnitude (F(1,32)=19.1, p<0.001), such that participants responded more quickly when saying 'yes' to large numbers and 'no' to small numbers. This pattern held across both even and odd numbers, and, indeed, there were no significant main effect of parity, nor interactions of parity with the other factors.

Discussion

Across two different tasks, numerical magnitude showed a significant interaction with verbal yes/no polarity: dominant responses ('yes') facilitated responses to large numbers,

non-dominant responses ('no') facilitated responding to small numbers. This result supports the polarity correspondence account of the SNARC effect, and demonstrates that spatial responses are not essential to the mapping of large numbers onto positive valence.

It is worth remembering that formally speaking, there is no reason for 'yes' responses to be faster, in general, than 'no' responses. In principle, in order to determine the parity of the number 4, one has to engage in the same process regardless of how one will subsequently respond to it. Nevertheless, the experiments performed here do not rule out an algorithmic level account for the observed 'yes' benefit. It may be, for instance, that people tend to engage in a confirmation strategy by (in essence) listing all the elements of the target "yes" set. So if 'yes' is mapped to 'even', then people might tend to evaluate the stimulus against the set 0, 2, 4, 6, 8. Since rejection would occur after comparison was done, acceptance (and consequently, 'yes' responses) would be relatively faster. Although Cho and Proctor (2007) argue that the stated task can influence which pole is dominant, it is still unclear why corresponding polarity should facilitate responding, and why dimensions have dominant values in the first place.

This experiment also demonstrated the known MARC effect for verbal tasks, but this is not terribly surprising. The effect of parity-task alignments was first demonstrated in a non-spatial context (Hines, 1990). In that instance, participants pressed a button (on the left) to designate that the two numbers were the 'same'; a right-hand button indicated 'different'. In that task, participants made 'same' responses more quickly to even than to odd numbers, in line with the polarity correspondence account. While the current work corroborates this result, it does not in our view contribute to it. What the current research provides is a compelling case that the influential mental number line theory requires something like the polarity correspondence
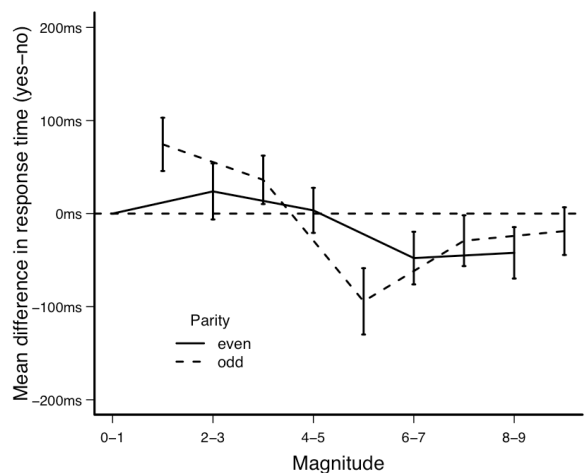

Figure 4: Mean difference between per-participant median response times for "yes" responses and "no" responses, collapsed across digit pairs, on the spoken magnitude task.

principle in order to explain compatibility effects involving the number line. The reverse is not obviously the case. This is made particularly strong by the fact that, while previous research has used polarity correspondence post-hoc to explain the SNARC effect (Proctor and Cho, 2006), the current research used the principle to predict a novel and otherwise surprising effect.

Nothing in either the polarity correspondence account or the current experiment is incompatible with the existence of a mental number line—it might be that numbers are somehow spatially encoded, and that this encoding is causally efficacious for some cognitive processes. However, the SNARC effect in most or all of its many manifestations does not seem to result from the mental number line. There are at least two ways that the conclusion 'SNARC implies mental number line' might be saved. The first is a general failure of the polarity correspondence principle. As mentioned in the introduction, the principle predicts not only the results reported here, but many others. Stimulus-response compatibility effects should be both ubiquitous, and more-or-less consistent. This might turn out not to be the case. If the general principle is falsified, both spatial and yes/no interactions with numerical magnitude will require special explanation. The only way to evaluate this possibility is to explore a range of predictions made by the general principle, and see whether other predictions are born out. Second, it might be that the mental number line makes predictions that are not already predicted by polarity correspondence. One potential instance is that it might be that the interaction between space and magnitude is stronger than the interaction between yes/no and magnitude (and that that difference cannot be accounted for by the generic strength of the polarity of the two dimensions). No such effect was found here: on the verbal response task, the mean benefit for compatible responses (i.e., saying 'yes' to large numbers, or 'no' to small ones) was 12ms; on the spatial response task, the mean compatibility benefit was 17ms, and the difference was not significant. However, such a difference might well exist. In that case, numbers might turn out to be especially spatial. In order to defend the SNARC effect as evidence for a specifically spatial representation of numbers, evidence needs to be produced that is not already predicted by the polarity correspondence account.

. Of course, one possible explanation for the compatibility effect shown here is that, like number, the affirmation dimension is *also* spatially coded, and the overlap between *these* codes yields the affirmation-numeric association. In our view, such a move is unmotivated and obfuscatory. If the effects reported in this study were generated by an implicit coding of "yes" as rightward, then we might expect an attenuation of the effect when compared to a literally leftward or rightward response. No such attenuation is immediately apparent. Previous experiments exploring stimulus-response compatibilities have generally used either a spatially varying stimulus dimension or a spatially varying response dimension (see Cho and Proctor, 2006 for an overview; though see Glass, Holyoak, and O'Dell, 1974). This experiment demonstrates that stimulus-response compatibilities typically attributed to that spatial component do not require explicitly spatial stimuli or responses

It is probably *true* that yes-responses and right-hand values are compatible (see Chase and Clark, 1971, for a vertical case). Indeed, the polarity correspondence principle predicts just this. However nothing in the experimental work or the existing literature indicates that representations of space are more fundamental than other representations. If every kind of response, and every kind of stimulus must be assumed to include a 'spatial' code, then the code in question is, we suggest, better thought of as not spatial, but as just a generic polarity. At the broadest level, then what is in question in this work is the fundamentality of space in abstract and formal reasoning.

Acknowledgments

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